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A METHOD FOR RAPID ESTIMATION OF MAXIMUM TANGENTIAL WIND SPEED IN TORNADOES [1]

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ABSTRACT

A widely applicable and rapidly computed method for estimating the maximum tangential wind speed in a tornado is developed. The method requires knowledge of the cloud deck height and the descent distance of the funnel cloud tip (often available from still photographs). The method is applied to the Dallas tornado of Apr. 2, 1957, and yields a maximum swirling speed of 209 mi hr^{-1}. Previous estimates for that whirlwind were 170 mi hr^{-1} (based on scrutiny of motion pictures of flying dust and debris) and 302 mi hr^{-1} (based on studying damage inflicted on structures and vehicles).

1. THE FUNNEL-CLOUD METHOD [2]

A steady inviscid axisymmetric flow is taken to describe a rapidly swirling flow above the thin surface frictional layer. If in inertial cylindrical polar coordinates the velocity

$$\mathbf{v}(r,z)=u(r,z)\,r+v(r,z)\Theta+w(r,z)\hat{z},$$

the radial and axial components of the momentum conservation equations are

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{1}$$

and

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g. \tag{2}$$

If the entropy is constant,[3] then $p=p(\rho)$; so it is convenient to define

$$P=\int\frac{dp}{\rho}. \tag{3}$$

Furthermore, if the secondary flow [4] is irrotational, that is, the vorticity $\boldsymbol{\omega}=\text{curl }\mathbf{v}=|\omega|\hat{z}$, then

$$v(r,z)\to v(r),\ \frac{\partial w}{\partial r}=\frac{\partial u}{\partial z}. \tag{4}$$

Then eq (1) and (2) admit the integral

$$P+\frac{u^2+w^2}{2}=-gz+\int\frac{v^2}{r}\,dr+\text{const.} \tag{5}$$

Since the moist adiabatic relation is analytically intractable, the convenient (and adequate) dry adiabatic relation for an ideal gas is adopted ($p\sim\rho^\gamma$):

$$c_pT+\frac{u^2+w^2}{2}=-gz+\int\frac{v^2}{r}\,dr+\text{const.} \tag{6}$$

Where the potential vortex model holds (i.e., where $vr\sim$ const), eq (6) becomes the isoenergetic relation

$$c_pT+\frac{u^2+v^2+w^2}{2}+gz=\text{const.} \tag{7}$$

However, attention is here concentrated on eq (6); the constant of integration is assigned by requiring $T\to T_0$, $u\to 0$, and $w\to 0$ as $z\to 0$ and $r\to\infty$. Then

$$c_p(T-T_0)+\frac{u^2+w^2}{2}=-gz+\int_\infty^r\frac{[v(r_1)]^2}{r_1}\,dr_1. \tag{8}$$

Once the secondary flow (u, w) and the primary flow (v) are specified, eq (8) describes any isotherm. In particular, the funnel cloud, which is the dew-point isotherm extended earthward from the cloud deck by rapid swirling, is described by eq (8) for $T=T_d$, the condensation temperature. The small variation of the dew point with pressure is here neglected.

Unfortunately, evidence concerning the secondary flow in most tornadoes is so sparse and uncertain that one usually cannot be certain whether a given tornado was of one-cell structure (radial inflow and axial updraft everywhere, with maximum updraft on the axis of rotation) or two-cell structure (axial downdraft and radial outflow near the axis, surrounded by a radial influx and axial updraft farther from the axis). The simplest resolution is to neglect the secondary flow; actually u^2 is almost certainly unimportant above the surface frictional layer, but dropping w^2 is less readily justified. In any case, eq (8) is

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[2] See also Dergarabedian and Fendell (1970).

[3] The approximations involved in the development of the method are clearly stated so the reader will be fully aware of them. The authors adopt some of the approximations for analytic convenience; they do not believe all involve excellent modeling of tornado conditions. Nevertheless, no approximation is felt to be poor; and the results upon application to data suggest that the arguments involved are reasonable. In particular, the theory yields condensation boundaries very similar to funnel-cloud shapes actually observed.

[4] The secondary flow is the streamline pattern described by the radial and axial velocity components.

here taken as[5]

$$c_p(T-T_0)\simeq -gz+\int_{\infty}^{r}\frac{[v(r_1)]^2}{r_1}\,dr_1. \tag{9}$$

The conservation of angular momentum is bypassed by adoption of the Rankine vortex model, a matching of rigid-body rotation near the axis to a potential vortex form far away from the axis:

$$v=\begin{cases} kr/2\pi a^2 & 0\leq r\leq a \\ k/2\pi r & a\leq r\leq \infty \end{cases} \tag{10}$$

where the circulation[6] approaches k as $r\to\infty$. As will be evident below, any other radial profile for the azimuthal velocity component may be adopted at this point if there is reason to believe it is more appropriate.

Substitution of eq (10) in eq (9) for $T=T_d$ gives, nondimensionally,

$$Z=1+\begin{cases} K\left(\frac{R^2}{2}-1\right) & R\leq 1 \\ -\frac{K}{2R^2} & R\geq 1 \end{cases} \tag{11}$$

where

$$Z=\frac{z}{c_p(T_0-T_d)/g},\ R=\frac{r}{a},\ \ K=\frac{k^2}{4\pi^2a^2c_p(T_0-T_d)}. \tag{12}$$

The quantity $[c_p(T_0-Td)/g]$, the vertical scale height, is the height of the ambient cloud deck and is henceforth denoted h. Actually, by analogy with aerodynamic wing theory, (h/a) is an "aspect ratio" for tornadoes; the larger this ratio, the better the axially invariant model for v holds over a wider range of z. The quantity K is the ratio of the square of maximum swirling speed to the potential energy associated with the cloud deck.

Comparison of eq (10) and (12) gives

$$v_{max}=\frac{k}{2\pi a}=[Kc_p(T_0-T_d)]^{1/2}=(Kgh)^{1/2}. \tag{13}$$

Equation (11) shows that, at $R=0$, K is that fraction of the distance from the ambient cloud deck ($Z=1$) to the ground ($Z=0$) to which the funnel cloud tip descends. Thus, for $K\leq 1$ (funnel-cloud tip part-way to ground or tangent to ground), eq (13) suffices; the maximum swirling speed is quickly calculated from knowledge of the tip height and cloud deck height. Here, for $K>1$ (finite funnel-cloud radius at the ground), K will be set to unity (although a more adequate procedure is easily devised). Of course, when the tip is close to the ground, it is often lost in dust and debris.[7]

The method will be applied to a particular tornado for illustrative purposes.

2. THE DALLAS TORNADO

On Apr. 2, 1957, the classical mid-American storm-generating pattern of low-level warm moist air from the south interacting with cold air from the west and northwest engendered 20 tornadoes over Oklahoma and Texas (Lee 1960). One of these twisters, which began about 4:30 p.m. CST over Dallas and lasted for about 34 min, is renowned because ". . . this tornado was observed in more detail than any other in history" (Beebe 1960). Extensive still- and motion-picture photography recorded the havoc wrought over a 16-mi path as the cyclonic tornado churned northward at about 27 mi hr^{-1} (Hoecker 1960*a*).

Hoecker has estimated the maximum swirling speed to have been at least 170 mi hr^{-1} on the basis of tracing debris and dust movement[8] recorded photographically (Hoecker 1960*b*).

Segner (1960) has attempted to estimate that wind speeds of at least 302 mi hr^{-1} were required to cause observed structural damage. Segner had to assume "reasonable sequences or modes of failure" and had to speculate "whether or not the damage resulted from actual wind forces or from flying objects" since the films did not show the destruction occurring. Segner examined nine events; he decided that winds of 217 mi hr^{-1} were required to effect eight of them. However, the ninth event (destruction of a billboard) required 302 mi hr^{-1} winds, although he notes that "the possibility of destruction from flying objects should not be overlooked because of the . . . proximity of other structures which were not severely damaged."

These estimates are now compared with results from the technique developed here, which uses photographs of the funnel-cloud length.

3. A NEW ESTIMATE OF MAXIMUM SWIRLING SPEEDS

Figure 1 presents data given by Hoecker (1960*a*) for the cloud deck height and the tip height, along with estimates of the maximum tangential speed computed from eq (13), as function of path length and time elapsed from initiation. One local maximum computed is 169 mi hr^{-1}

[5] Dropping w^2 results in a small underestimate of funnel cloud descent distance and hence a conservative estimate of maximum swirling speed.

[6] The circulation is the scalar defined by the line integral of fluid velocity around a closed curve. Here, the curve is most readily taken as the circle $r=r_1>a$, z=const.

[7] The funnel-cloud photograph may also be used to estimate a, introduced in eq (10). The maximum swirling speed occurs at $r=a$, that is, $R=1$; thus, the locus of the maximum swirling speed is a right-circular cylinder. This cylinder intersects the funnel cloud, according to eq (11), in the circle $R=1$, $Z=1-(K/2)$. But for $0<K\leq 1$, eq (11) also shows that the funnel cloud extends from $Z=1$ (where $R\to\infty$) to $Z=1-K$ (where $R=0$). Thus $Z=1-(K/2)$, the height at which the radius of the funnel cloud is the radius where the swirling speed is maximum, is also the height at which the axial distance from the ambient cloud deck to the funnel cloud tip is bisected. Since knowledge of the ambient cloud deck height provides a scale for a photograph, often the photograph can be used to obtain a good approximation to a. From knowledge of K [see the sentence just below eq (13)] and of $[c_p(T_0-T_d)]$ [see the sentence just below eq (12)], the photograph then provides all the data required to find k [from the definition of K given in eq (12)].

[8] Hoecker's analysis has been scrutinized by Morton (1966) regarding proper accounting for the discrepancy between forces on debris and forces on fluid particles.

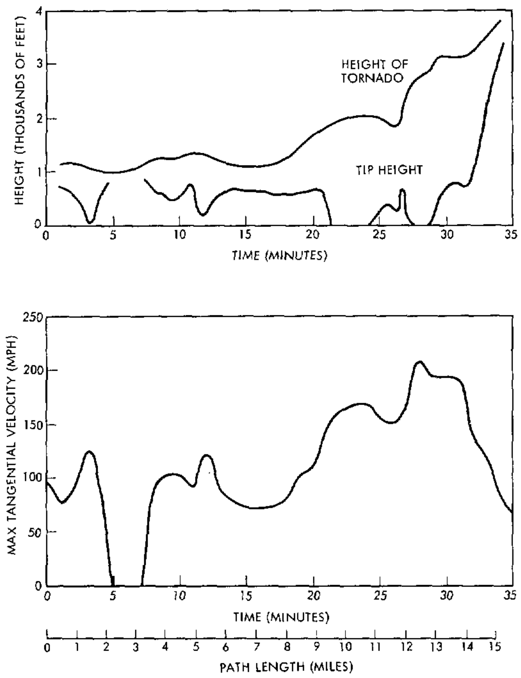

FIGURE 1.—Hoecker's data for the cloud deck height and the descent distance of the funnel-cloud tip is plotted against the computed maximum tangential speed for the entire 34-min lifetime of the tornado. The correlation of path length from the point of first sighting with time was given by Hoecker.

at the 24- to 25-min mark; this magnitude is close to the 170 mi hr^{-1} maximum found by Hoecker (1960*b*). Incidentally, intensive destructiveness began at the 22-min mark and persisted for the next 10 min (Beebe 1960); this is the interval of high speeds according to the calculations. However, figure 1 indicates an absolute maximum of 209 mi hr^{-1} at the 29-min mark. Interestingly, the billboard, the destruction of which led Segner (1960) to estimate a maximum speed of 302 mi hr^{-1}, was passed by the tornado at about the 29-min mark; furthermore, Segner's second highest estimate (deduced from the overturning of a railroad car) of 217 mi hr^{-1} occurred at the 25-min mark along the path.

It should be noted that a quasi-steady approximation is being invoked in that a steady-state theory is being used to describe a temporally changing sequence of states.

In conjunction with two points raised earlier in the paper, it might be worth noting that Hoecker (1960*b*) suggests that the Dallas tornado may have been of one-cell structure at lower altitudes and of two-cell structure at greater altitudes. However, the point is not firmly resolved. Also, Hoecker (1960*b*) shows that the radial profile of the tangential velocity is approximated very well by the Rankine vortex at an altitude of 1,000 ft. At lower altitudes, Hoecker finds that the tangential velocity component does not decay with the inverse first power of the cylindrical radial coordinate. The departure from potential vortex form may well be owing to surface frictional effects.

REFERENCES

Beebe, Robert G., "The Life Cycle of the Dallas Tornado," *U.S. Weather Bureau Research Paper* No. 41, Washington, D.C., **1960**, pp. 3–52.

Dergarabedian, Paul, and Fendell, Francis, "On Estimation of Maximum Wind Speeds in Tornadoes and Hurricanes," *The Journal of the Astronautical Sciences*, Vol. 17, No. 4, Jan.–Feb. **1970**, pp. 218–236.

Hoecker, Walter H., Jr., "The Dimensional and Rotational Characteristics of the Tornadoes and Their Cloud System," *U.S. Weather Bureau Research Paper* No. 41, Washington, D.C., **1960*a***, pp. 53–113.

Hoecker, Walter H., Jr., "Wind Speed and Air Flow Patterns in the Dallas Tornado of April 2, 1957," *Monthly Weather Review*, Vol. 88, No. 5, May **1960*b***, pp. 167–180.

Lee, J. T., "The Synoptic Situation," *U.S. Weather Bureau Research Paper* No. 41, Washington, D.C., **1960**, pp. 114–142.

Morton, B. R., "Geophysical Vortices," *Progress in Aeronautical Science*, Vol. **7**, Pergamon Press, New York, **1966**, pp. 145–194.

Segner, P., Jr., "Estimates of Minimum Wind Forces Causing Structural Damage," *U.S. Weather Bureau Research Paper* No. 41, Washington, D.C., **1960**, pp. 169–175.

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